

Movie Scheduling for Multiplexes

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Introduction

Decisions need to be made every day in movie theatres: which movies to screen, where and when to screen them.

In the past, theatres have mostly relied on intuition; existing procedures do not ensure the generation of optimal solutions and are lacking in efficiency.

Our aim: Formulate a linear integer program to efficiently generate a feasible movie schedule which maximizes a theatre's profit.



Background & Literature

- Directed graphs – nodes represent movies being played at a specific theatre at some specific time, directed edge points to another theatre and time at which the movie can be played (Eliash et al.)
- Linear integer programs with objective function and constraints (SilverScreener algorithm, Swami et al.)

Model Formulation



Variables

1. $\{m_1, \dots, m_M\}$ set of M different movies
2. $\{t_1, \dots, t_N\}$ set of N time periods
3. $\{s_1, \dots, s_S\}$ set of S screens (theatre rooms)

Variables (Cont.)

1. p_j capacity of the room with screen j
2. b_i time period during which movie i begins playing
3. $r_{i, j, k}$ revenue generated by playing movie i on screen j during the k -th time period after i 's first screening
4. POP_i popularity of movie i
5. RC_i rental cost of movie i for a single time period
6. OC operational cost (constant) of theatre for a single time period

Decision Variables

1.
$$x_{i,j,k} = \begin{cases} 1 & \text{if } m_i \text{ is played on } s_j \text{ during the } k\text{-th time period after } m_i\text{'s first screening} \\ 0 & \text{otherwise} \end{cases}$$

2.
$$y_{i,k} = \begin{cases} 1 & \text{if } x_{i,j,k} = 1 \text{ for at least one } j \\ 0 & \text{otherwise} \end{cases}$$

Assumptions

1. We rent a movie right after it is released; there is no time lag between a rental placement and its arrival
2. The logistical costs and the operation costs can be estimated
3. The demand or popularity of a movie can be projected and measured

Constraints

- 1.** $\sum_{i \in [M]} x_{i, j, t-b_i} \leq 1 \forall j \in [S], t \in [N]$ 2 movies cannot be played on the same screen at a given time
- 2.** $\sum_{j \in [S]} x_{i, j, k} \leq S \cdot y_{i, k} \forall i \in [M], k \in [N]$ ensures that the two binary variables are in synch
- 3.** $\sum_{j \in [S]} x_{i, j, k} \leq p \cdot S \forall i \in [M], k \in [N]$ "diversity" constraint

Objective Function

$$TP = TR - TC$$

We aim to maximize total profit (TP), which we will formulate as the difference between total revenue (TR) and total cost (TC)

Total Revenue Formulation

$$TR = \sum_{i, j, k} x_{i, j, k} \cdot r_{i, j, k}$$

$$r_{i, j, k} = \frac{POP_i \cdot c_1 \cdot p_j}{c_2 \cdot k}$$

- We assume that revenue is **positively** correlated with the popularity of a movie and the capacity of a room; and **negatively** related to the time for which a movie has been released
- We later provide a machine learning approach to project the popularity of a movie

Total Cost Formulation

$$TC = \sum_{i \in [M]} \sum_{k \in [N]} RC_i \cdot y_{i,k} + N \cdot OC$$

Total cost (TC) consists of renting costs (RC) for movies and constant operating costs (OC) for each time period that the theatre is open

* Our objective is to maximize profit -> minimize cost ->

$y_{i,k}$ will be zero whenever possible

Model Formulation

$$\max \left(\sum_{i, j, k} x_{i, j, k} \cdot r_{i, j, k} - \left(\sum_{i \in [M]} \sum_{k \in [N]} RC_i \cdot y_{i, k} + N \cdot OC \right) \right)$$

$$\text{s.t.} \quad \sum_{i \in [M]} x_{i, j, t-b_i} \leq 1 \quad \forall j \in [S], t \in [N] \quad x_{i, j, k} \in \{0, 1\}$$

$$y_{i, k} \in \{0, 1\}$$

$$\sum_{j \in [S]} x_{i, j, k} \leq S \cdot y_{i, k} \quad \forall i \in [M], k \in [N]$$

$$\sum_{j \in [S]} x_{i, j, k} \leq p \cdot S \quad \forall i \in [M], k \in [N]$$

A black and white photograph of a vintage movie camera. The camera is shown from a side-on perspective, with a film strip running through its mechanism. The camera has several large gears and a lens. The background is a plain, light color. The text "Predicting Popularity" is overlaid on the camera in a white, serif font.

Predicting Popularity

Predicting Popularity

Figure 1: Distribution of IMDB Rating

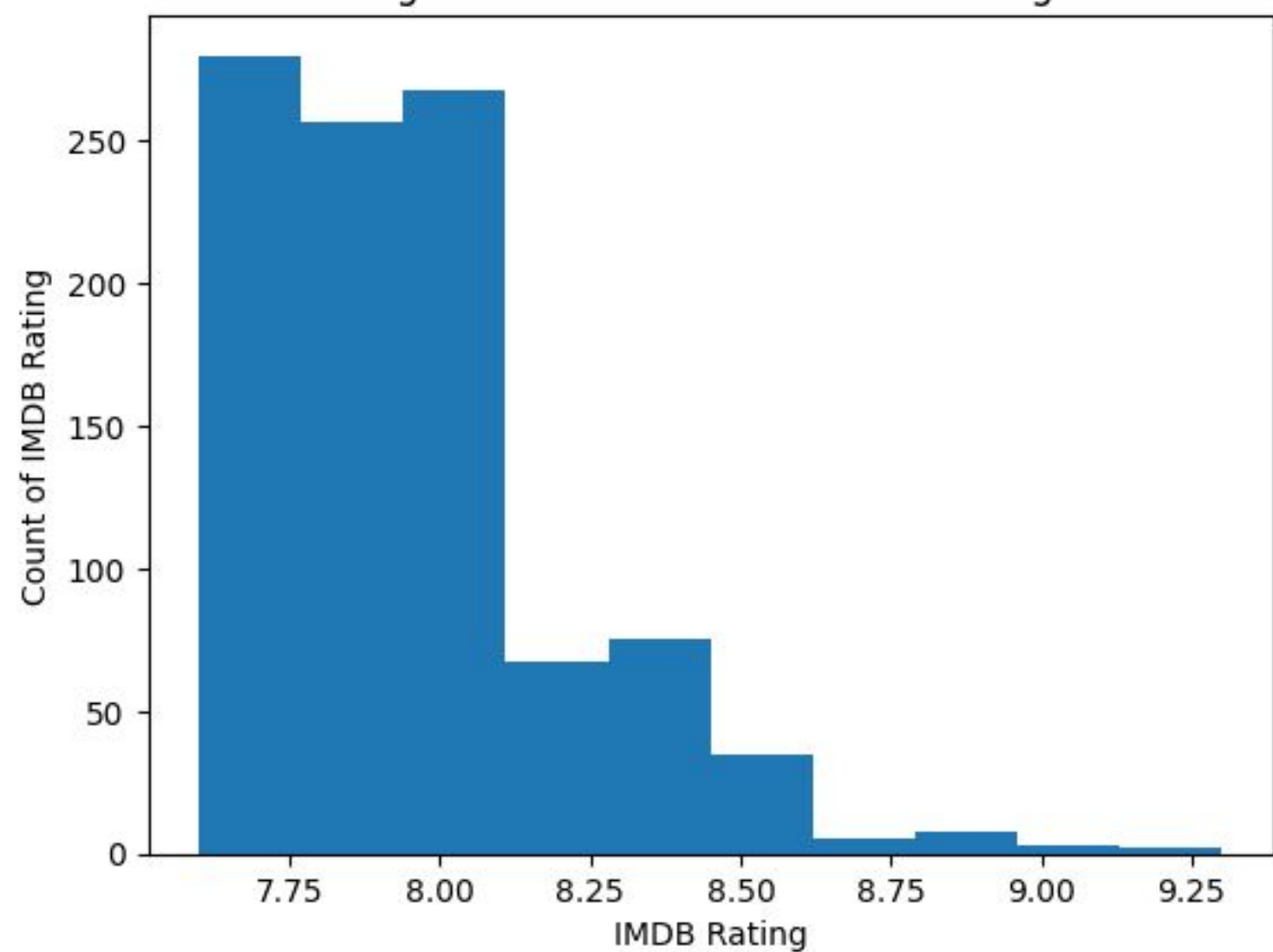
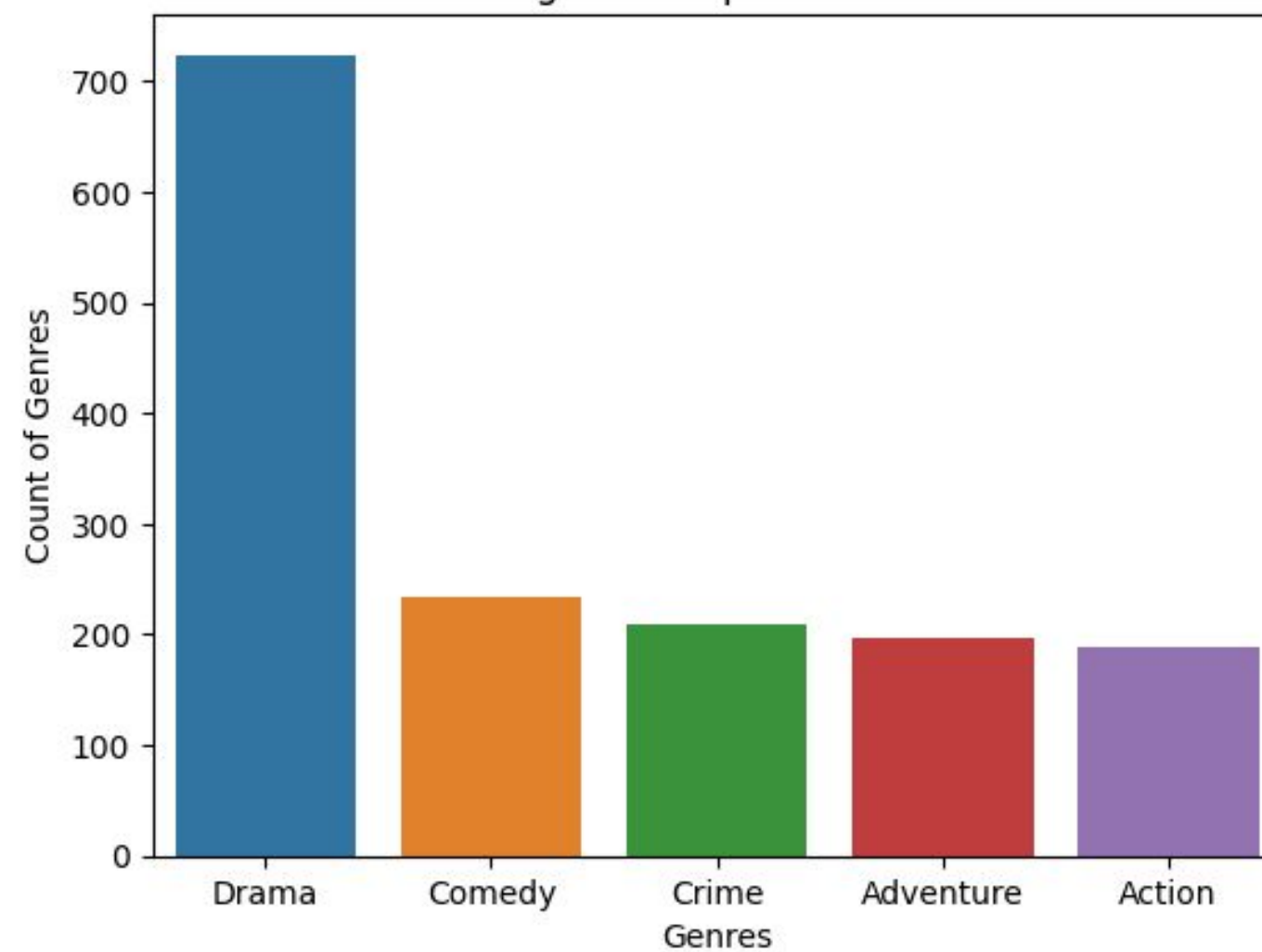
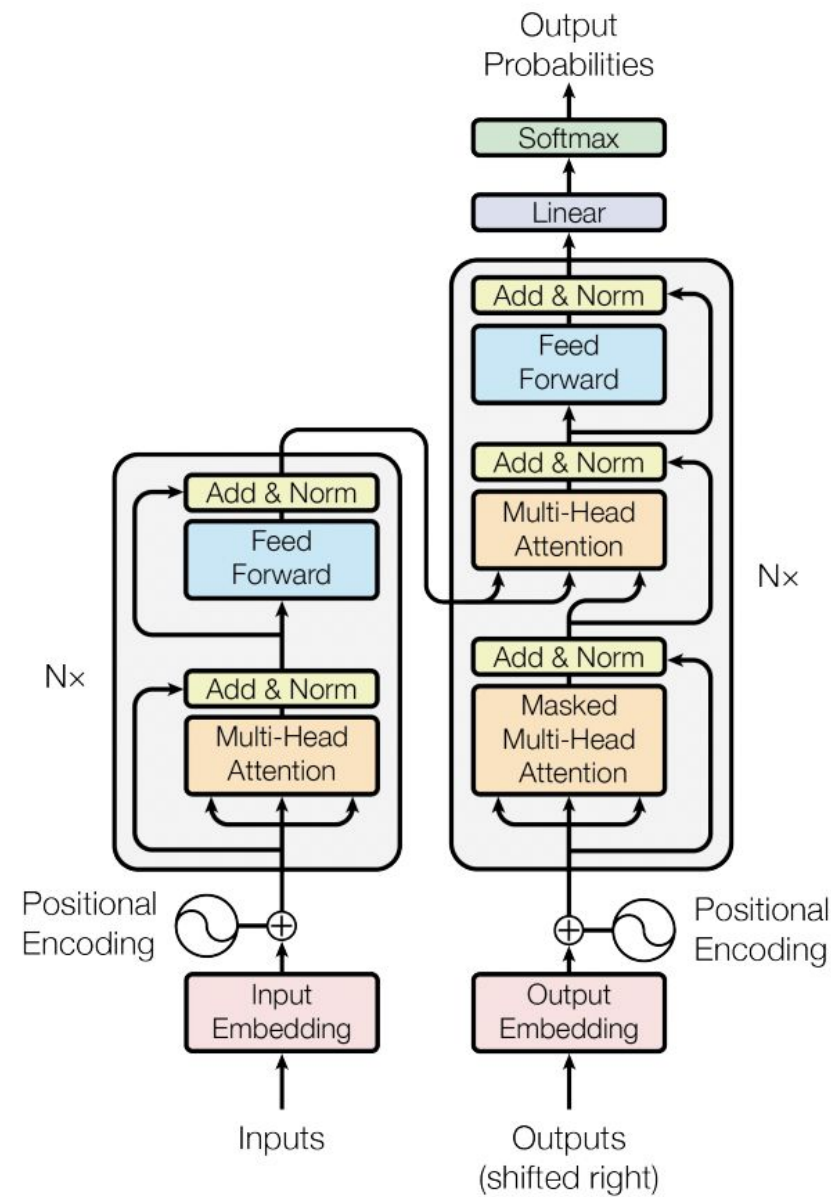


Figure 2: Top 5 Genres



Predicting Popularity



R-Squared of All Models	
Model	R-Squared
Linear Regression	-0.1827
Neural Network	-3.1798
SVM	0.0748

The background of the image consists of several black and white film strips that are curled and overlapping. The strips are arranged in a way that creates a sense of depth and movement, with some strips in the foreground being sharper than others in the background. The overall aesthetic is that of a classic film reel.

Case Study

Assumptions

1. We assume that the theatre is open 24/7, and have our time periods be two-hour blocks
2. We start every movie on the hour, and we assume each movie lasts 2 hours
3. Each movie is released at exactly 0:00 on its release date
4. The theatre is filled for every screening during the week
5. Our movie theatre consists of 6 screens, 3 small-sized (50 seats) and 3 larger-sized (100 seats)
6. Operational cost per period is $\$20,000/N$
7. We restrict our attention to a one-week time frame

Data



Release date: July 21 (let this be t_1)
Movie ticket price: \$30
Movie rental price: \$20,000 per week



Release date:
July 21
Movie ticket
price: \$20
Movie rental
price: \$15,000
per week

Release date: July 23
Movie ticket price:
\$10
Movie rental price:
\$5,000 per week



Gurobi Solution

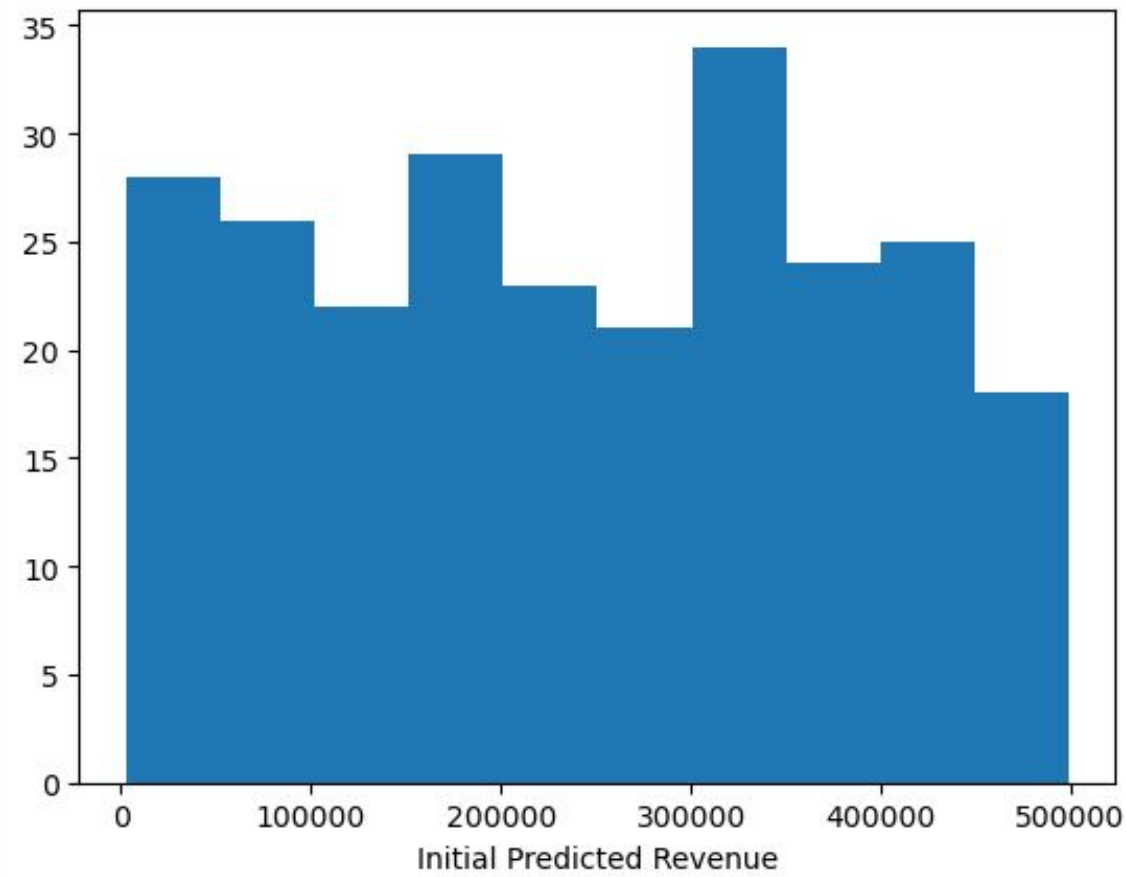
We show the schedule generated for screen 3 by Gurobi below
Overall objective value: \$185,967

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day7
0:00-2:00	Barbie	Barbie	Oppenheimer	Oppenheimer	Spirited Away	Spirited Away	Spirited Away
2:00-4:00	Barbie	Barbie	Oppenheimer	Oppenheimer	Spirited Away	Spirited Away	Spirited Away
4:00-6:00	Barbie	Barbie	Spirited Away	Spirited Away	Spirited Away	Spirited Away	Spirited Away
6:00-8:00	Barbie	Barbie	Spirited Away	Spirited Away	Spirited Away	Spirited Away	Spirited Away
8:00-10:00	Barbie	Barbie	Spirited Away	Spirited Away	Spirited Away	Spirited Away	Spirited Away
10:00-12:00	Barbie	Barbie	Spirited Away	Spirited Away	Spirited Away	Spirited Away	Spirited Away
12:00-14:00	Barbie	Barbie	Spirited Away	Spirited Away	Spirited Away	Spirited Away	Spirited Away
14:00-16:00	Barbie	Barbie	Oppenheimer	Spirited Away	Spirited Away	Spirited Away	Spirited Away
16:00-18:00	Barbie	Barbie	Oppenheimer	Spirited Away	Spirited Away	Spirited Away	Spirited Away
18:00-20:00	Barbie	Barbie	Oppenheimer	Spirited Away	Spirited Away	Spirited Away	Spirited Away
20:00-22:00	Barbie	Barbie	Oppenheimer	Spirited Away	Spirited Away	Spirited Away	Spirited Away
22:00-24:00	Barbie	Oppenheimer	Oppenheimer	Spirited Away	Spirited Away	Spirited Away	Spirited Away

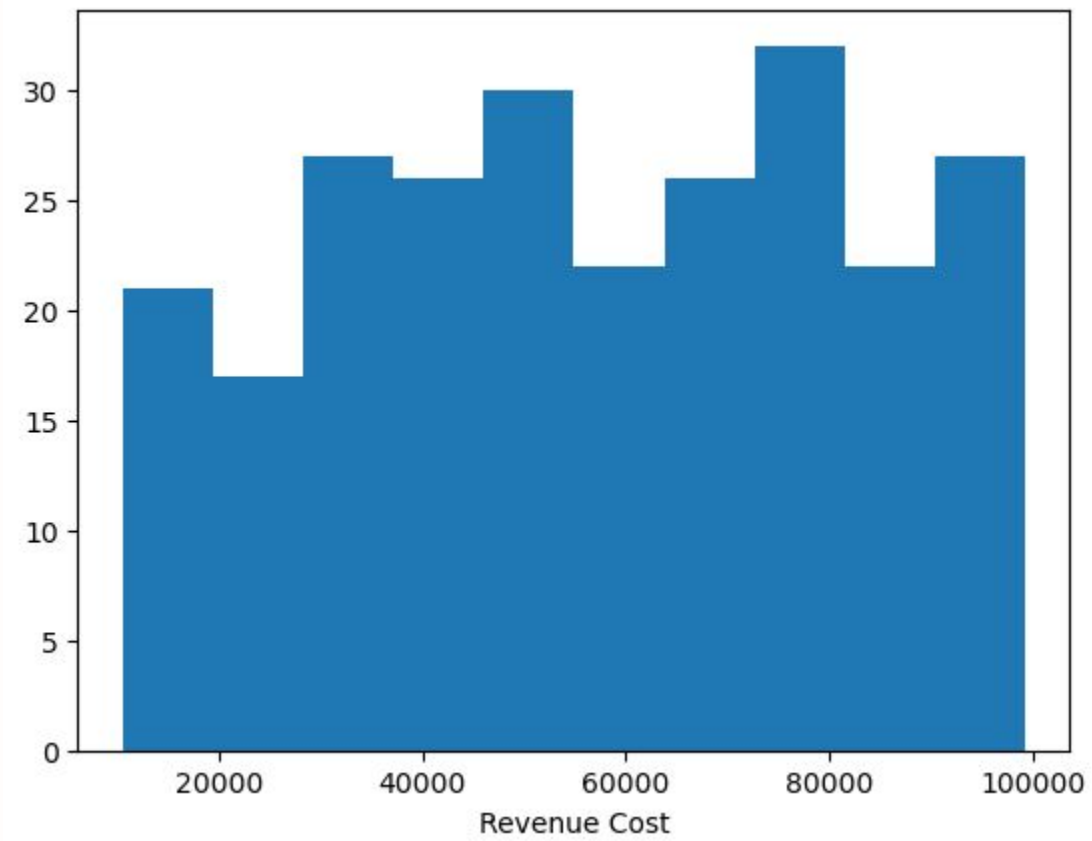


Experimentation

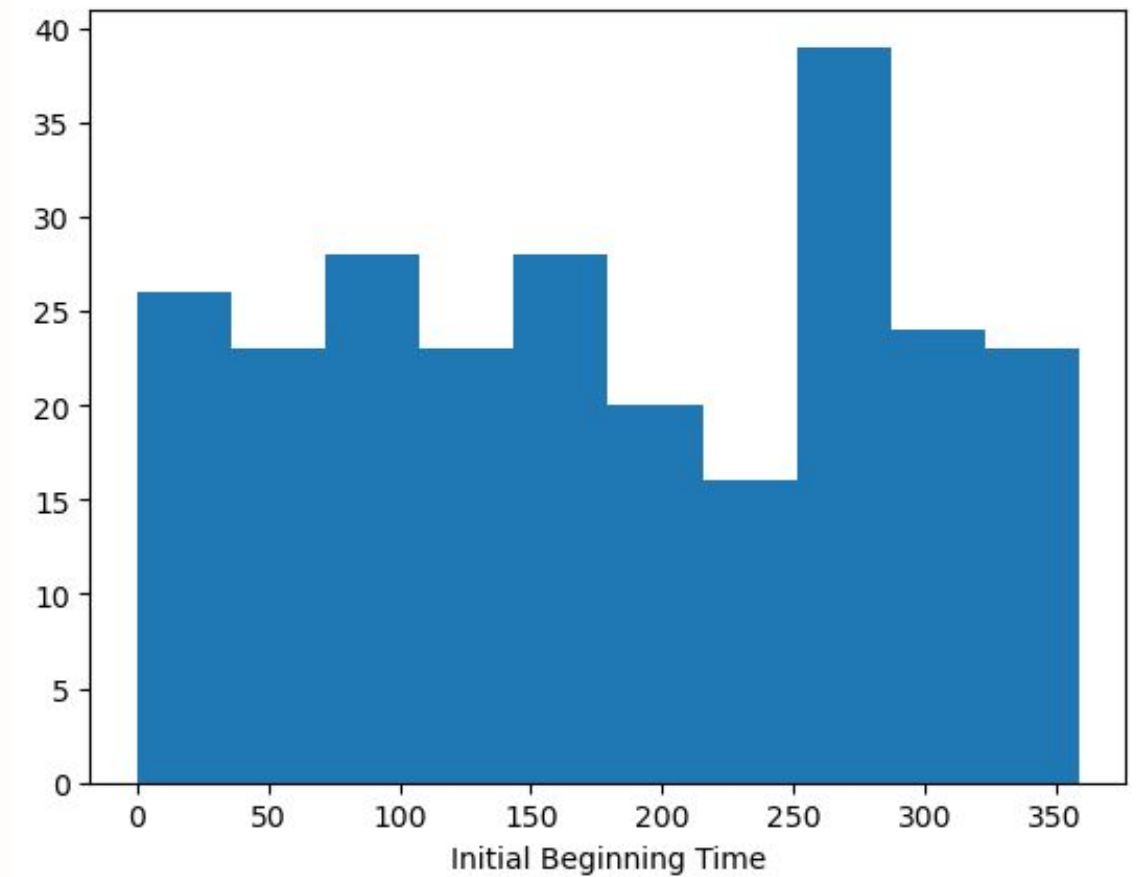
Experimentation



\$2,000 ~ \$500,000



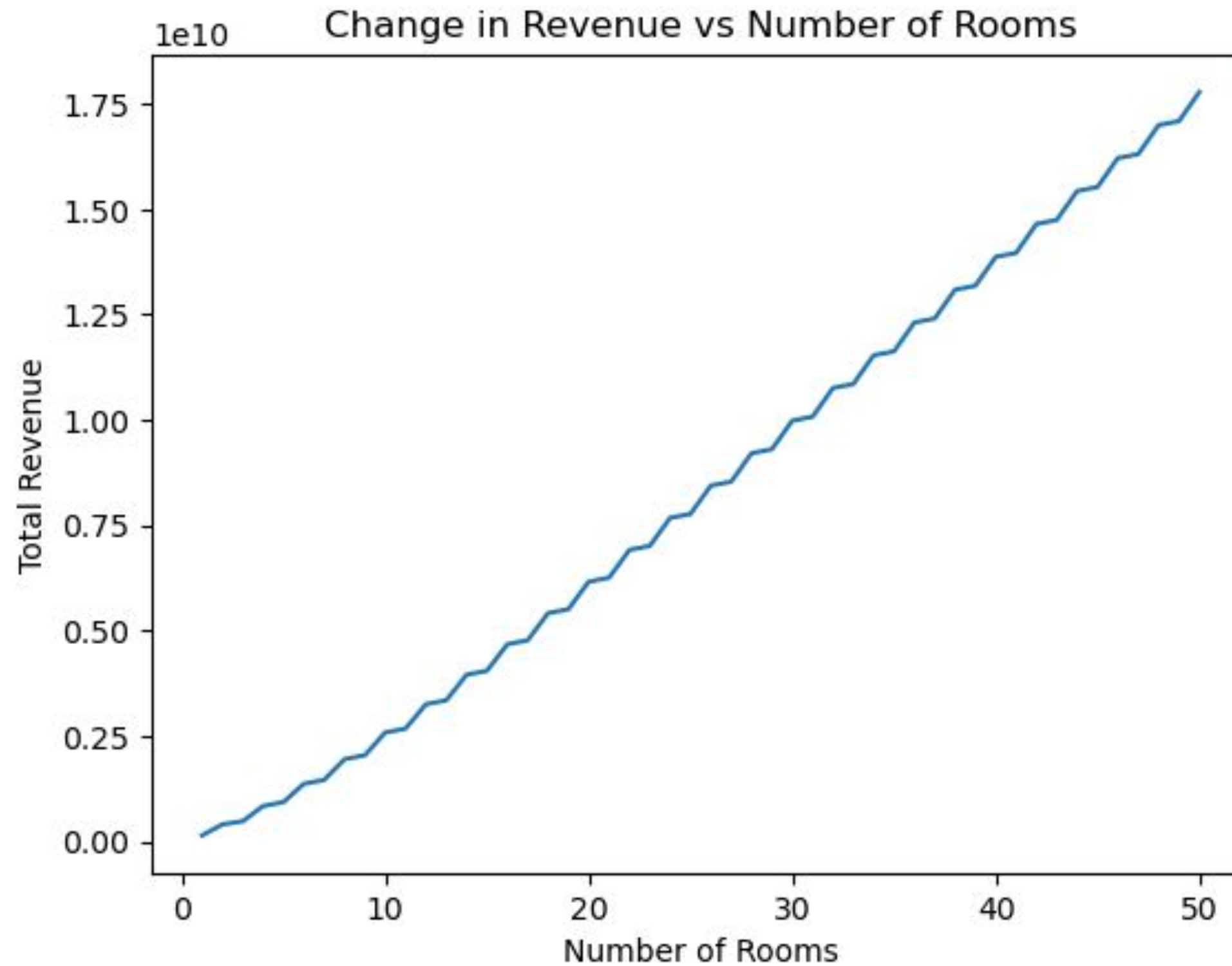
\$10,000 ~ \$100,000



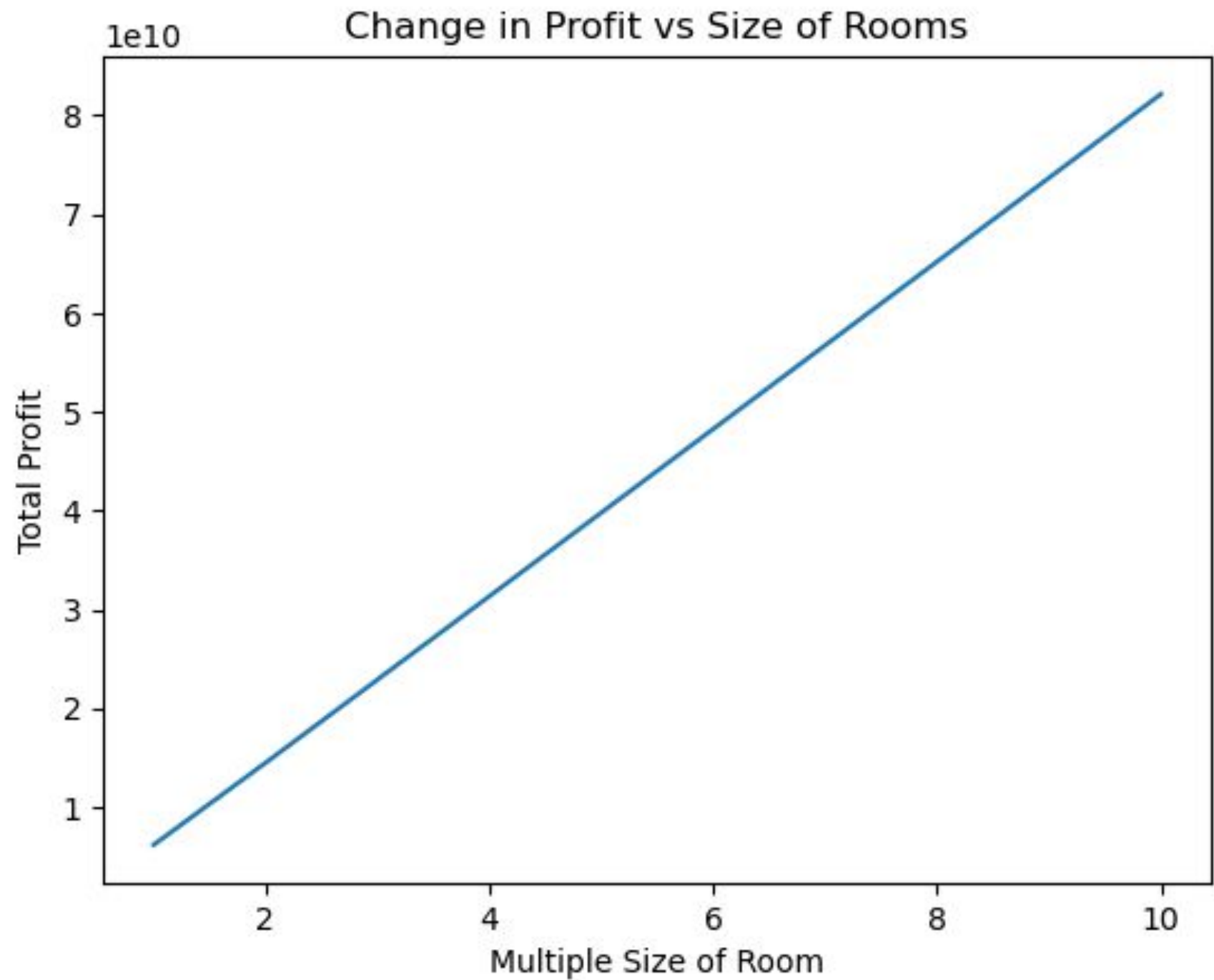
1 ~ 360

M = 100, N = 360

Experimentation

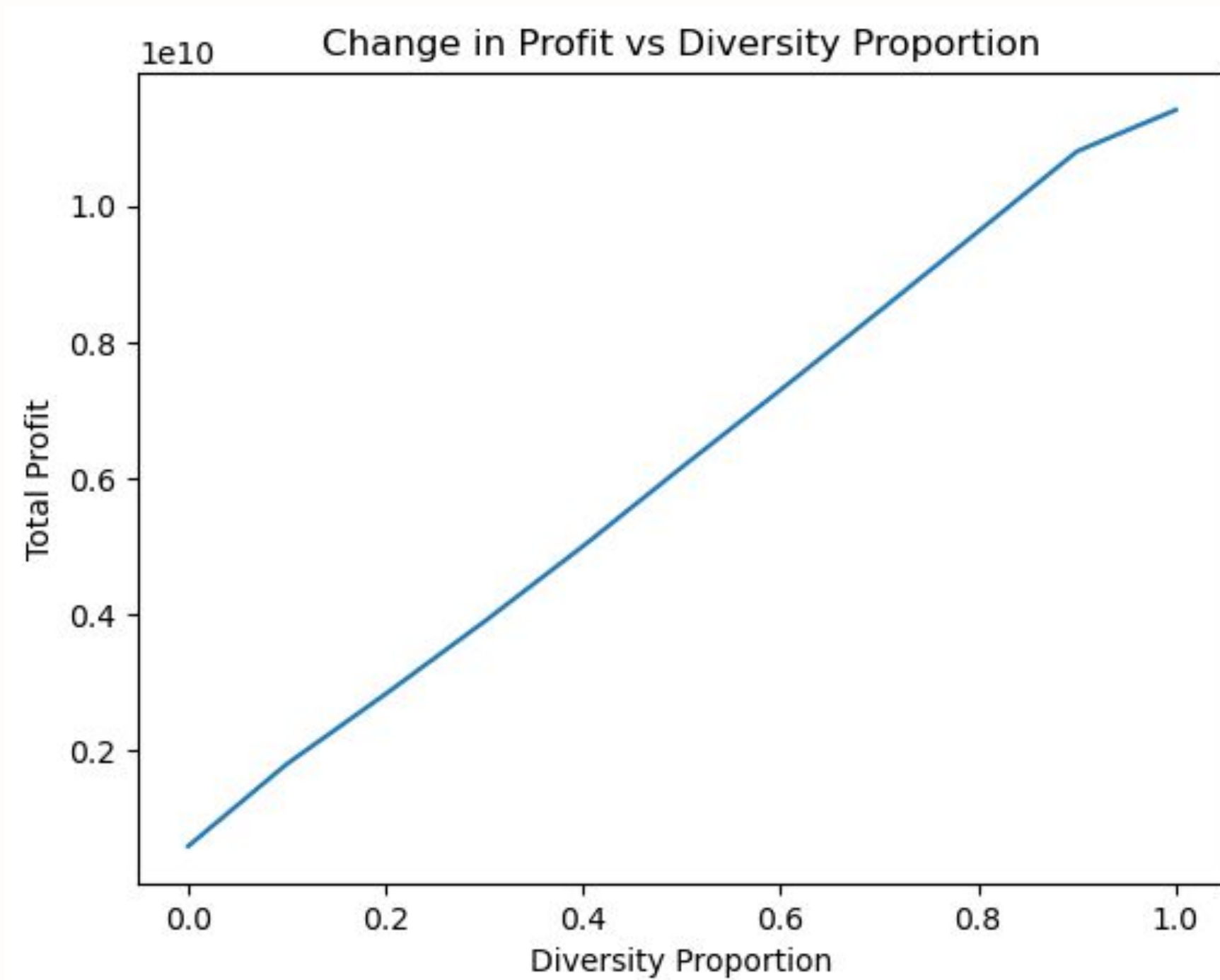


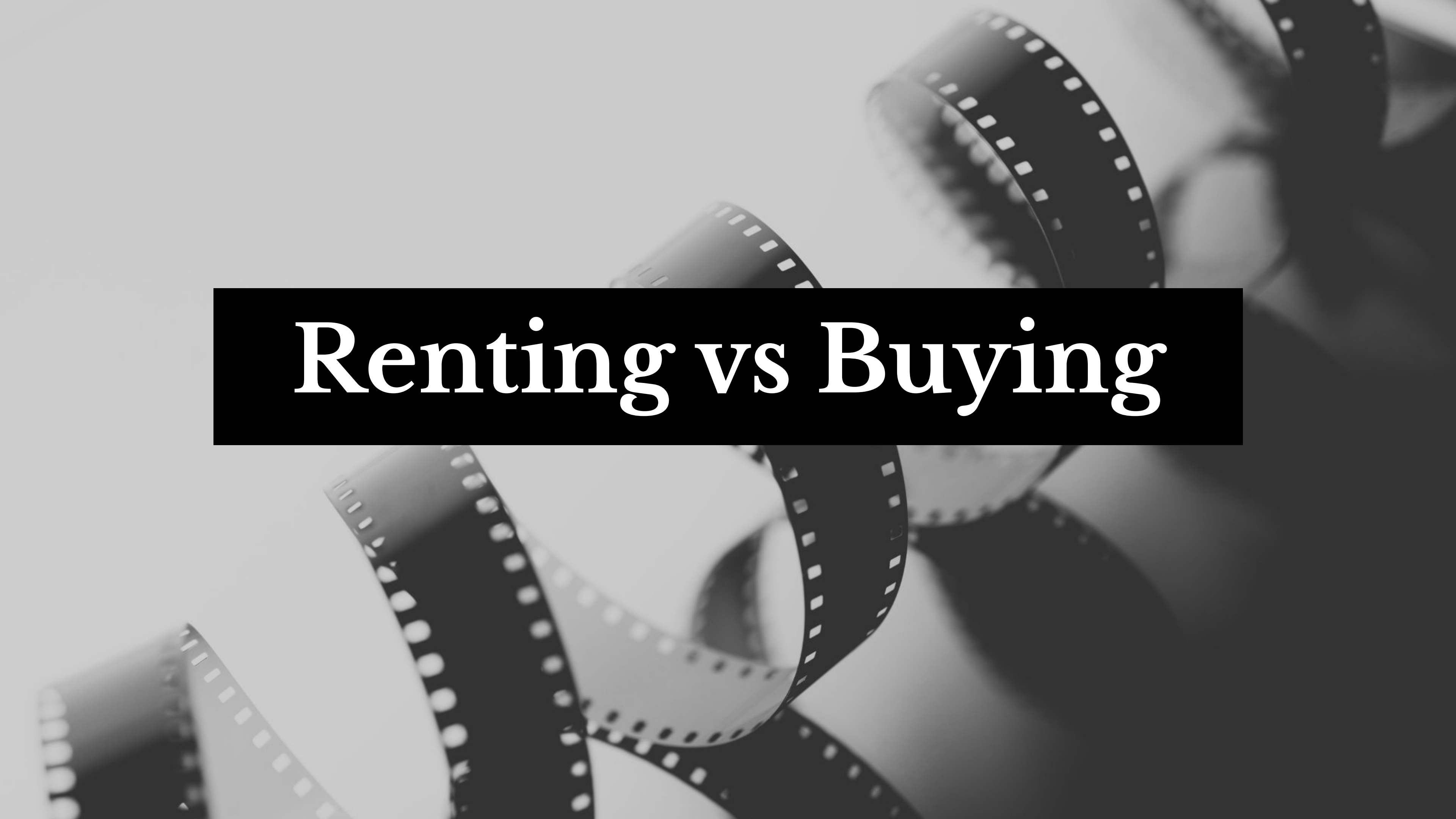
Experimentation



	Profit
Doubling Half the Rooms	\$13,613,686,490
1.5x All the Rooms	\$10,275,598,545

Experimentation





Renting vs Buying

Renting vs Buying

$$\max \sum_{i,j,k} x_{i,j,k} r_{i,j,k} - \left(\sum_{i \in [M]} \text{BC}_i \cdot z_i + N \cdot \text{OC} \right)$$

$$\sum_{i \in [M]} x_{i,j,k} \leq 1, \quad \forall j \in [S], t \in [N]$$

$$\sum_{i \in [M]} \sum_{j \in [S]} x_{i,j,k} \leq S \cdot N \cdot z_i \quad \forall k \in [N]$$

$$\sum_{j \in [S]} x_{i,j,k} \leq \lfloor S/2 \rfloor \quad \forall i \in [M], k \in [N]$$

$$x_{i,j,k} \in \{0, 1\}$$

$$y_{i,k} \in \{0, 1\}$$

Renting vs Buying

	Profit	Number of Unique Movies
Renting	\$5,726,053,327	228
Buying	\$4,767,736,137	114



Limitations

Limitations of the Model

- Popularity and cost can only be estimated; values may deviate
- How to select set of movies is not included
 - In reality, there are many movies that could be played
 - But we assume we are given a set of M movies to be scheduled
- Movie theatres normally split box office revenue with studios which produce movies
 - The profit predicted by our model is higher than the actual profit received by the theatre
- Sources of revenue are not exhaustive; other sources of revenue include:
 - Retail & merchandise sales
 - Individual rental services
- Influencing factors are not exhaustive
 - Time of day
 - Season
 - Holidays



Thank You!

Any questions?